Continuous and discrete numerical modeling: a comparison for flood inundation mapping

Silvia Di Francesco, Niccolò Cusano University, silvia.difrancesco@unicusano.it
Sara Venturi, University of Perugia, sara.venturi@unipg.it
Chiara Biscarini, UNESCO Water Chair, Foreigners University of Perugia, chiara.biscarini@unistrapg.it
Lucio Ubertini, H2CU – Sapienza University, Rome, lucio.ubertini@uniroma1.it

Continuous and discrete Lattice Boltzmann numerical modelling of shallow water equations

The different computational performances and possible points of contact between the approach based on continuous modelling (Navier-Stokes) and the discrete one at the mesoscopic scale (LBM models - Lattice Boltzmann Methods) have been investigated.

In this work some outcomes related to the validation of the models solving the **shallow water equations** (SWE) solved with LBM and also with the classical continuous approach to Navier Stokes equations are presented and then some results concerning the performance of the simulations are shown. The solution of shallow water equations by using the LBM approach was firstly due to Zhou (2004). Also Tubbs (2010) and Geveler (2010) contributed in an innovative way for developing the scientific research about Lattice Boltzmann model for shallow water equations. The equations for solving SWE using LBM are:

- LBM streaming and collision equations: $f_\alpha(\mathbf{x}+\mathbf{e}_\alpha \Delta t, t+\Delta t) - f_\alpha(\mathbf{x}, t) = \Omega_\alpha$
- Macroscopic values for depth h and velocity v: $\sum_{\alpha=1}^{b-1} f_\alpha = h \quad \sum_{\alpha=1}^{b-1} f_\alpha e_\alpha = h\mathbf{v}$
- BGK Collision Operator (Bhatnagar et al., 1954): $\Omega_\alpha = -\frac{1}{\tau}\left(f_\alpha - f_\alpha^{eq}\right)$

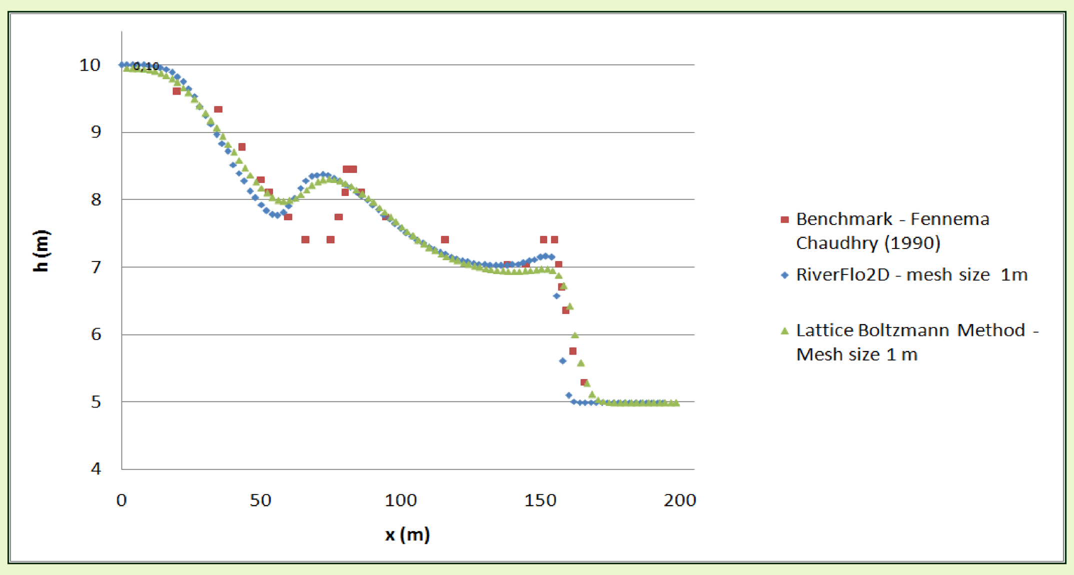
In the **graph 1** and in the **figure 1, 2** and **3** a comparison between the results of Fennema & Chaudhry dam-break obtained by the continuum model and by the LBM model are presented. In particular, the **graph 1** compares the water surface levels at the cross section in the centre of the break with the ones of the classical benchmark problem. The results obtained in the two models are comparable with each other and very close to the Fennema & Chaudhry dam break.

In **table 1, 2** and in **graph 2, 3, 4** some results about the computational efficiency are presented. In particular, the **graph 2** displays the difference in simulation time of discrete and continuous model showing that, for the same mesh dimension, the LB simulation has a computational velocity significantly higher.

The **graphs 3, 4** describe in the two models the value of the time of a cycle (time needed to perform a cycle) for one node as function of the number of nodes. The LBM and RiverFlo 2D models have a behavior absolutely different. In the LBM model the value of the time of a cycle increases with the number of the nodes. Instead, in the RiverFlow2D model, the value of this parameter firstly decreases, then it remains almost constant to the exceeding of a threshold value (about 2000000 nodes) of the number of nodes.

In the **figures 4, 5, 6, 7, 8** the scientific study carried out on behalf of the Province of Arezzo (Italy) on the floodplain of the Cerfone River at Mercatale is presented. The hydraulic simulation was performed by using the RiverFlo2D model. In **figure 5** the triangular mesh used in the hydraulic simulation is shown. In **figure 6, 7, 8** the extension of flooding areas after 3.5, 4.5 and 5.5 hours from the start of the flood hydrograph is presented. As shown, the model appears to schematize the effective trends of the flood, even if it uses high computational times.

Model validation by benchmark problems: Fennema & Chaudhry Dam Break (1970)



Graph 1: Water surface at cross-section a-a. Comparison between results of continuum and discrete model

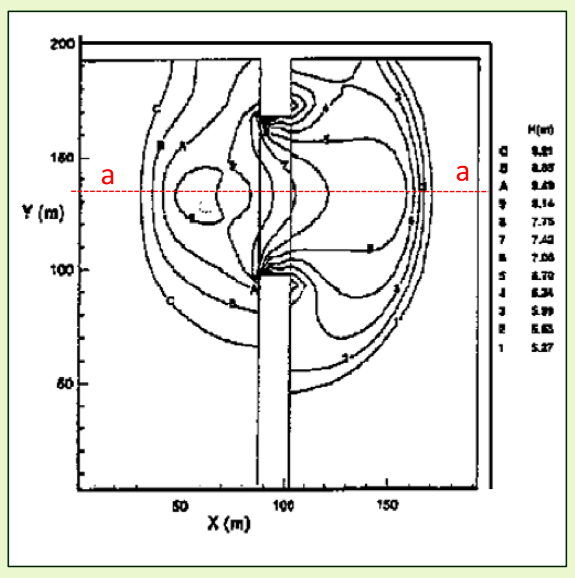


Fig 1: Fennema & Chaudhry Dam Break contours

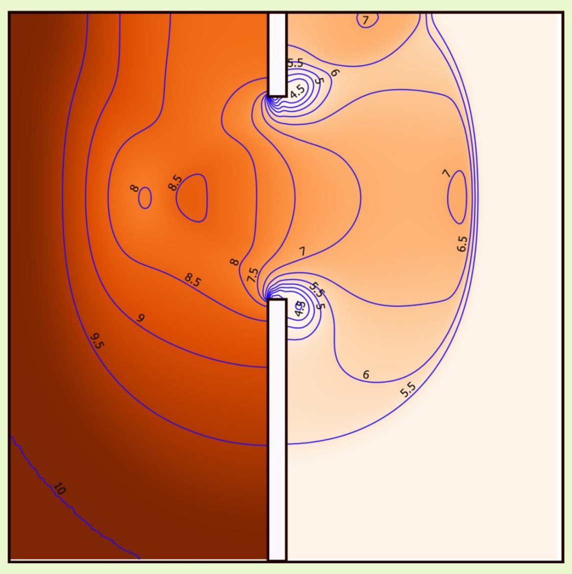


Fig 2: Water surface contours – RiverFlo 2D Model

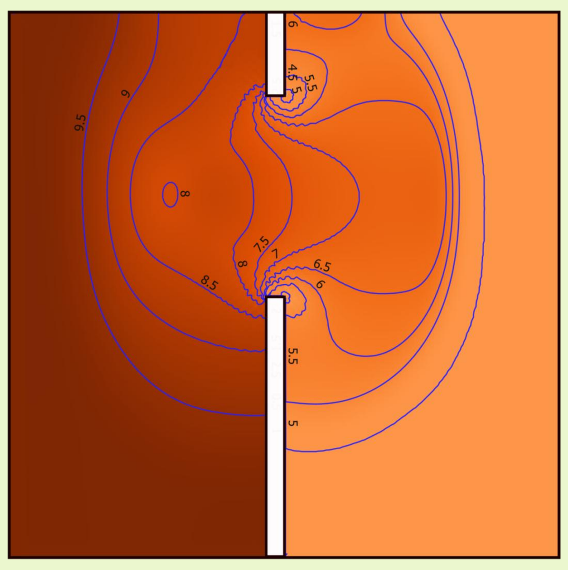


Fig 3: Water surface contours – LBM Model

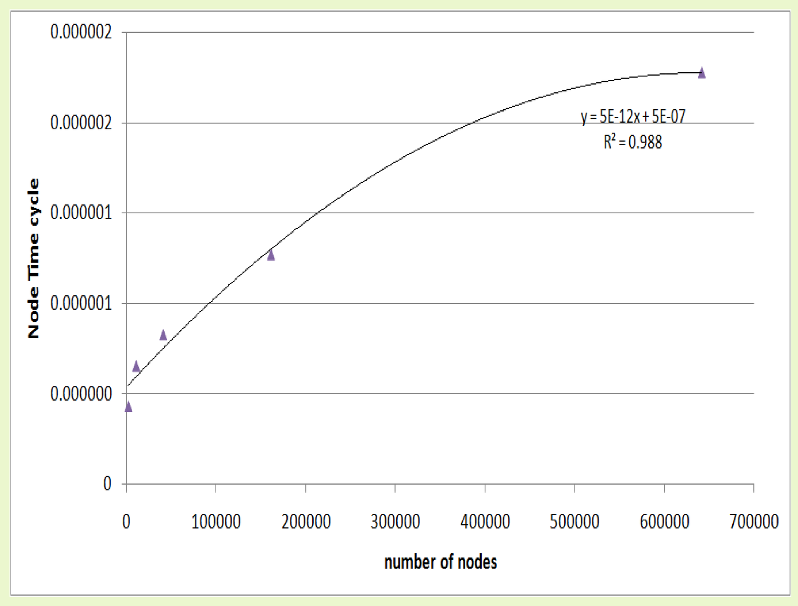
Discrete Lattice Boltzmann Model v.s. Continuous Model RiverFlo 2D Computational Efficiency

Discrete Modelling - LBM						
		mesh size				
		5	2	1	0.5	0.25
Simulation Time	sec	10.4047	95.8580	479.9580	2937.2356	21014.8493
Number of Nodes	-	1681	10201	40401	160801	641601
Cicles per sec	-	1384.0927	150.2327	30.0047	4.9029	0.6853
Cicle per sec/nodes	-	8.2337E-01	1.4727E-02	7.4267E-04	3.0491E-05	1.0681E-06
Time cycle	sec	3.4722E-04	1.3889E-04	6.9444E-05	3.4722E-05	1.7361E-05
Time cycle /nodes	sec	2.0656E-07	1.3615E-08	1.7189E-09	2.1593E-10	2.7059E-11

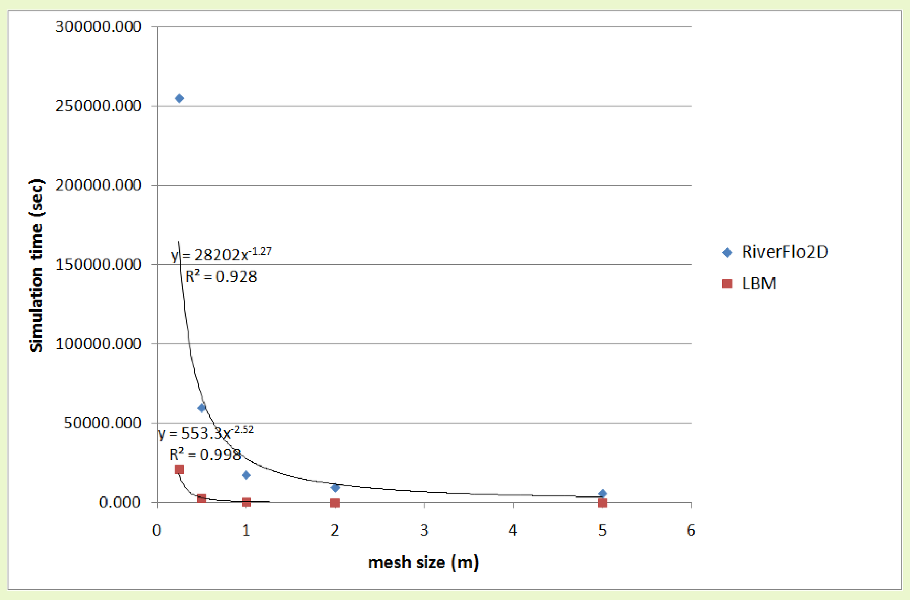
Table 1: Dam break simulation Lattice Boltzmann model

Continuous Modelling RiverFlo-2d						
		mesh size				
		5	2	1	0.5	0.25
Simulation Time	sec	5513	9302	17213	59643	255074
Number of Nodes	-	1933	11722	46251	183225	731506
Cicles per sec	-	2.6120	1.5481	0.8366	0.2414	0.0565
Cicle per sec/nodes	-	1.3513E-03	1.3206E-04	1.8088E-05	1.3177E-06	7.7175E-08
Time cycle	sec	3.8285E-01	6.4597E-01	1.1953E+00	4.1419E+00	1.7714E+01
Time cycle /nodes	sec	1.9806E-04	5.5108E-05	2.5845E-05	2.2605E-05	2.4215E-05

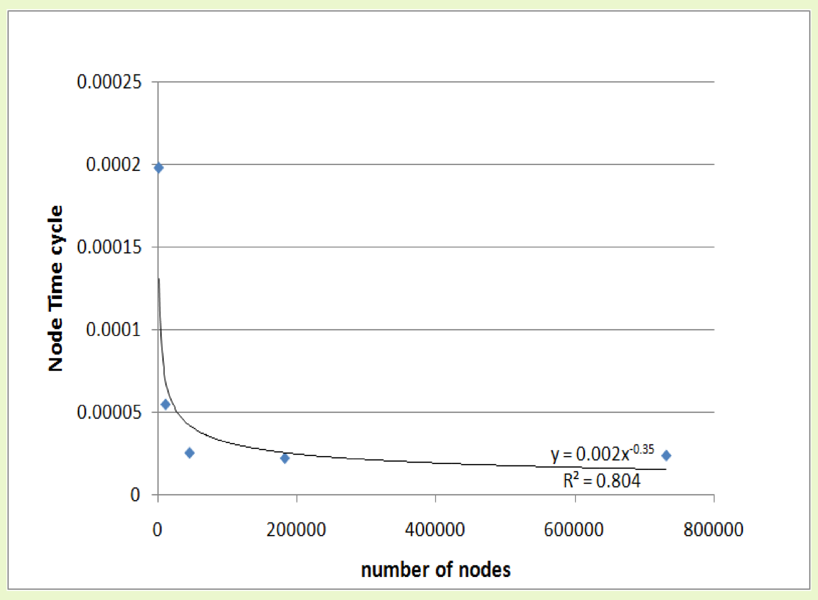
Table 2: Dam break simulation River Flo 2D Model



Graph 3: NodeTime cycle as function of the number of nodes– Lattice Boltzmann Model



Graph 2: Simulation time in a logarithmic scale as function of mesh size



Graph 4: Node Time cycle as function of number of nodes– River Flo 2D Model

Computational Modeling of Cerfone River at Mercatale (Tuscany, Italy)

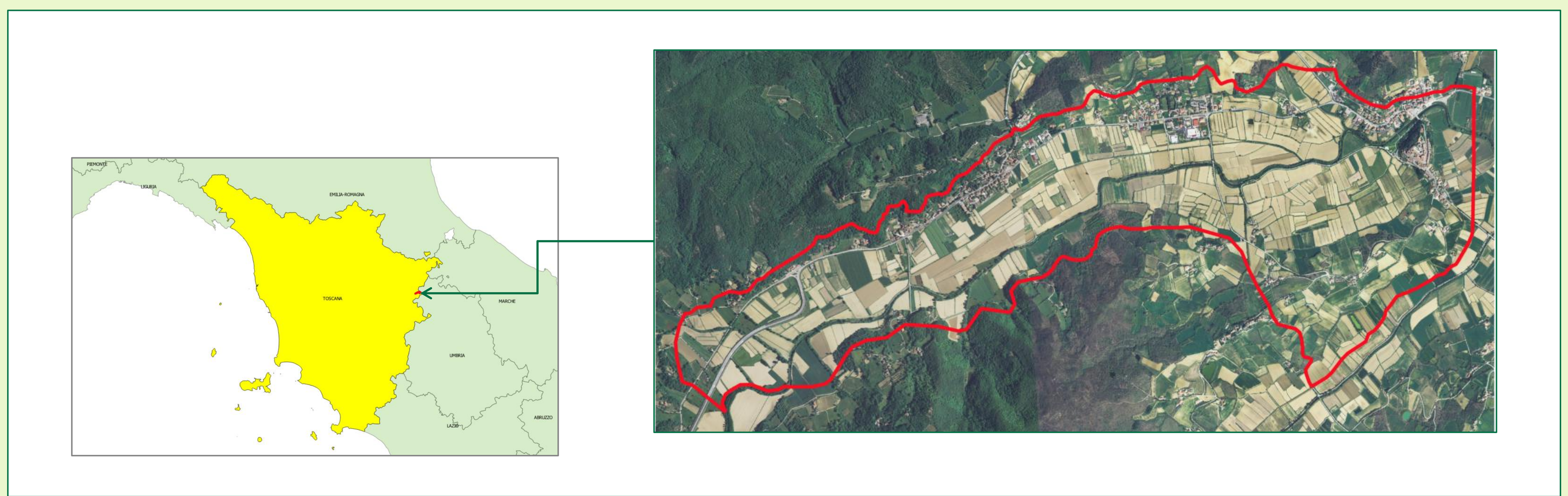


Fig 4: RiverFlo 2D Cerfone Model – geographic location

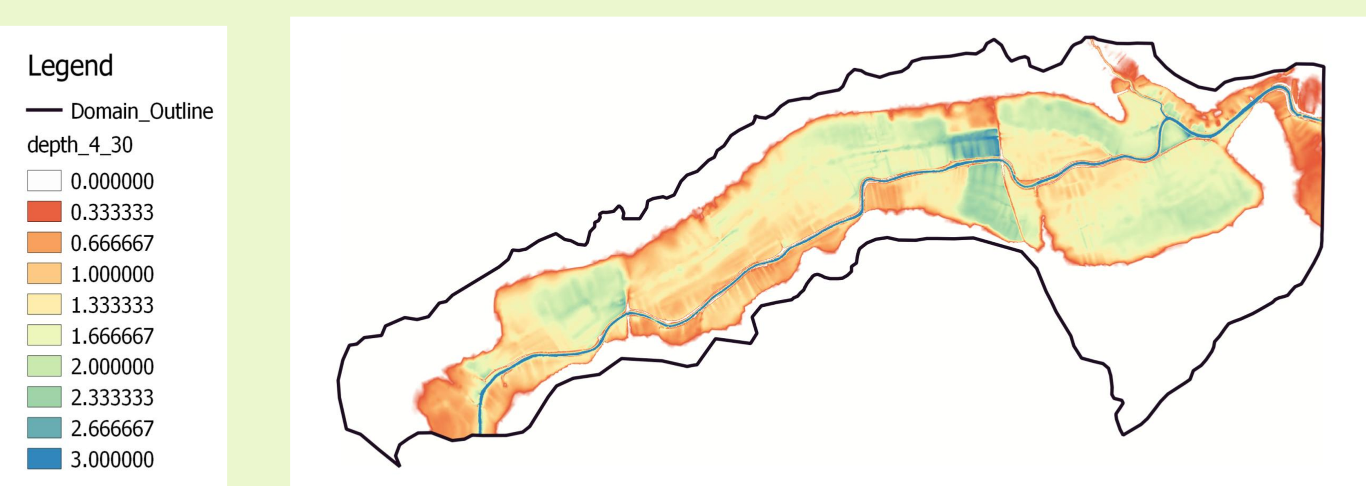


Fig 7: Flooding map showing the water depth – 4.5 h

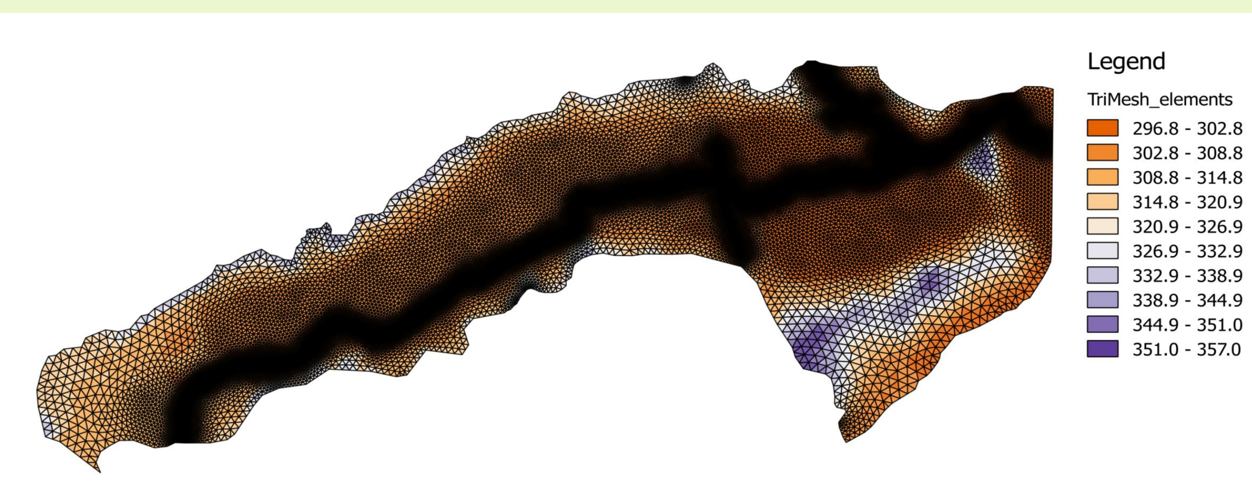


Fig 5: RiverFlo 2D triangular mesh in the simulation domain

Fig 6: Flooding map showing the water depth – 3.5 h

Fig 8: Flooding map showing the water depth – 5.5 h