Continuous and discrete Lattice Boltzmann numerical modelling of shallow water equations

The different computational performances and possible points of contact between the approach based on continuous modelling (Navier-Stokes) and the discrete one at the mesoscale level (LBM models - Lattice Boltzmann Methods) have been investigated.

In this work some outcomes related to the validation of the models solving the shallow water equations (SWE) solved with LBM and also with the classical continuous approach to Navier Stokes equations are presented and then some results concerning the performance of the simulations are shown. The solution of shallow water equations by using the LBM approach was firstly due to Zhou (2004), also Tubbs (2010) and Geveler (2010) contributed in an innovative way for developing the scientific research about Lattice Boltzmann model for shallow water equations. The equations for solving SWE using LBM are:

- LBM streaming and collision equations: $f_x(x, t + \Delta t) - f_x(x, t) = \Omega_x$
- Macroscopic values for depth $h$ and velocity $v$: $\sum f_x = h \sum f_x v_x = hv$
- BGK Collision Operator (Bhatnagar et al., 1954): $\Omega_x = -\frac{1}{2}(f_x - f_x^e)$

In the graph 1 and in the figure 1, 2 and 3 a comparison between the results of Fennema & Chaudhry dam-break obtained by the continuum model and by the LBM model are presented. In particular, the graph 1 compares the water surface levels at the cross section in the centre of the break with the ones of the classical benchmark problem. The results obtained in the two models are comparable with each other and very close to the Fennema & Chaudhry dam break.

In table 1, 2 and in graph 2, 3, 4 some results about the computational efficiency are presented. In particular, the graph 2 displays the difference in simulation time of discrete and continuous model showing that, for the same mesh dimension, the LB simulation has a computational velocity significantly higher.

The graphs 3, 4 describe in the two models the value of the time of a cycle (time needed to perform a cycle) for one node as function of the number of nodes. The LBM and RiverFlo 2D models have a behavior absolutely different. In the LBM model the value of the time of a cycle increases with the number of the nodes. Instead, in the RiverFlow2D model, the value of this parameter firstly decreases, then it remains almost constant to the exceeding of a threshold value (about 200000 nodes) of the number of nodes.

In the figures 4, 5, 6, 7, 8 the scientific study carried out on behalf of the Province of Arezzo (Italy) on the floodplain of the Cerfone River at Mercatale is presented. The hydraulic simulation was performed by using the RiverFlo2D model. In figure 5 the triangular mesh used in the hydraulic simulation is shown. In figure 6, 7, 8 the extension of flooding areas after 3.5, 4.5 and 5.5 hours from the start of the flood hydrograph is presented. As shown, the model appears to schematize the effective trends of the flood, even if it uses high computational times.

**Graph 1: Water surface cross-section at a comparison between results of continuum and discrete model**

**Graph 2: Simulation time in a logarithmic scale as function of number of nodes - LBM model**

**Graph 3: Node Time cycle as function of number of nodes - Lattice Boltzmann Model**

**Graph 4: Node Time cycle as function of number of nodes - RiverFlo 2D Model**

**Figure 3: Water surface contours - RiverFlo 2D Model**

**Figure 4: Water surface contours - LBM Model**

**Table 1: Dam break simulation Lattice Boltzmann model**

**Table 2: Dam break simulation RiverFlo 2D Model**

**Figure 5: RiverFlo 2D triangular mesh in the simulation domain**

**Figure 6: Flooding map showing the water depth – 3.5 h**

**Figure 7: Flooding map showing the water depth – 4.5 h**

**Figure 8: Flooding map showing the water depth – 5.5 h**

**Continuous and discrete flooding inundation mapping**

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